Eulerian formulation of inelasticity – from metal plasticity to growth of biological tissues

MB Rubin
Faculty of Mechanical Engineering
Technion - Israel Institute of Technology
Haifa, Israel 32000
Email: mbrubin@tx.technion.ac.il

Abstract

The Lagrangian formulation of metal plasticity by Bilby et al. (1957), Kroner (1959) and Lee (1969) is based on the total deformation gradient $\mathbf{F}$ from a fixed reference configuration and a plastic deformation measure $\mathbf{F}_p$ from the reference configuration to a stress-free intermediate configuration. The elastic deformation can be defined by $\mathbf{F}_e = \mathbf{F}\mathbf{F}_p^{-1}$. Besseling (1966) proposed an evolution equation for $\mathbf{F}_e$. Eckart (1948) proposed an Eulerian evolution equation directly for an elastic deformation measure to model elastically isotropic elastic-inelastic material response. Leonov (1976) developed the same theory for polymeric liquids. Motivated by this work, Rubin (1994) proposed Eulerian evolution equations for a triad $\mathbf{m}_i$ of microstructural vectors that model elastic deformation for fully anisotropic elastic-inelastic response. In contrast with the Lagrangian formulation, this Eulerian formulation is insensitive to arbitrariness of the choices of: a reference configuration, an intermediate configuration, a total deformation measure and a plastic deformation measure. For elastically isotropic response, Eulerian evolution equations are proposed for the elastic dilatation J_e and a unimodular elastic distortional deformation tensor $\mathbf{B}'_e$. For growth of biological tissues, the evolution equations have been modified to model homeostasis, which is the inelastic process that causes a tendency for $J_e, \mathbf{B}'_e$ to approach their homeostatic values $J_h, \mathbf{H}'$. In particular, the stress in the homeostatic state can be non-zero. Constitutive equations must be proposed for both the homeostatic values $J_h, \mathbf{H}'$ and the rates Γ_m, Γ of dilatational and distortional homeostasis, respectively. Robust, strongly objective numerical algorithms will also be discussed.